The Observatory, **127**, No 1, 53–59
February 2007

FIRST VISIBILITY OF THE LUNAR CRESCENT: BEYOND DANJON'S LIMIT

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Many methods for predicting lunar first visibility have been proposed throughout history and new models are still being developed. All these models have to be compared with the published observations to test their validity.

In this paper, we use our photometric model to predict the naked-eye first visibility of the lunar crescent. We find that an elongation of about 7.5° is the lowest naked eye visibility limit. We also find that lunar crescent may be seen –with a suitable telescopic magnification and ideal local conditions- when the Moon is about 5° from the Sun. Consequently, the thin lunar crescent may be seen in a telescope even at new Moon when the Moon is at its greatest inclination.

Introduction

Methods for predicting first sighting of the new crescent moon have been around since the time of the Sumerians. Efforts for obtaining an astronomical criterion for predicting the time of first lunar visibility go back to the Babylonian era, with significant improvements and work done later by ancient Hebrew and Muslim scientists and others.

In the twentieth century Fotheringham [1], Maunder [2], Danjon [3], Bruin [4], McNally [5], Schaefer [6], Ilyas [7], Yallop [8], Fatoohi [9], Caldwell [10] and recently Odeh [11] have developed empirical methods for predicting first sighting of the new crescent moon. Each of them made a collection of observations of first visibility of the crescent and used statistical analysis to find a minimum elongation for the Moon to be first seen: 12° for Fotheringham, 11° for Maunder, 10-10.5° for Yallop and Ilyas[9] , 7.5° for Fatoohi , 7° for Danjon and Schaefer, 6.4 ° for Odeh and 5° for McNally.

After testing all empirical models, Doggett and Schaefer have claimed the model by Schaefer yielded significantly better prediction than any other algorithms [12]. Fatoohi and his colleagues at the University of Durham examined the Danjon limit by using 52 observations (with elongations ≤ 9.4°) extracted from 503 ancient and modern observations of first and last visibility of the lunar crescent. They deduced that Danjon's 7° limit should actually be revised to 7.5°.

In effect, within their empirical methods, Bruin, Schaefer and Yallop have all tried to take into consideration of some physical aspects concerning prediction of first lunar visibility. In this paper, we are following their steps, benefiting from new technologies and recent researches, especially in eye perception[13-15].

Parameters to be considered

For evaluating a physical (photometric) method that predicts the first visibility of the lunar thin crescent the factors to be considered are: (*i*) The geometry of the Sun, the Moon, and the horizon, (*ii*) the width and luminance of the crescent, (*iii*) the absorption of moonlight by the atmosphere, (*iv*) the scattering of light in the atmosphere, and (*v*) the psychophysiology of the human vision. The first three factors can be calculated easily. Factor (*iv*) depends on the local conditions and we are obliged to take these measurements[16]. Factor (*v*)has been discussed in one of our earlier works[17].

Discussion

As we have mentioned previously, prediction of the first sighting of the early crescent moon is a complex problem because it involves a number of highly non-linear variables simultaneously. It is very popular to use the age of the Moon (i.e. the time of sighting since the instant of new moon) to predict if the young crescent will be visible. The naked-eye record (15.0 hours) set on 1990 February 25 by John Pierce[18] is still unbroken. However, some times even a 30- hour-old crescent can remain invisible if the Moon is low in the sky at sunset, and well to the north or south of the sunset point. For this reason, we consider in our calculations the perfect geometrical situation when the difference in azimuth between the Sun and the Moon at the moment of observation equals zero, DAZ = 0°.

By measuring the twilight sky luminance and considering our site elevation, we fix the local atmospheric conditions. Moreover, we have calculated the optimum lunar altitude for first visibility[16], so the only variable still to be considered is the lunar elongation. We know that the Moon can pass directly in front of the Sun at new Moon (when a solar eclipse occurs) or can pass as far as five degrees away. That is, the Moon can start the (lunar) month with an elongation ranging from zero to about five degrees. To explore the idea that the Danjon limit might actually be smaller than 7˚, on one hand, and to investigate the possibility of spotting the thin lunar crescent at new Moon when the Moon is at its greatest inclination on the other hand, we calculate the visibility criteria when the Moon is at the two extreme cases; near apogee and near perigee.

Naked-eye visibility limit calculations

We calculate the *Visibility Criteria* using our photometric model equations[19]; Blackwell's 1946 extensive data[20], our site elevation, and our local brightness of the background sky (See an illustrative example in reference 16). We perform these calculations starting with the phase angle of 175˚ (this angle corresponds to a topocentric elongation of 4˚) and incrementing 0.5˚ in each step of calculations (possible error = ±0.25˚). The phase angle of 175˚ coincides with the instant of the new moon on March 14, 2002 when the Moon was near apogee (Table I). In the second case, when the Moon is near perigee, the moon attained that angle 8 hours after the new moon of2003 November 23; in this case the instant of the new moon occured during a total solar eclipse (Table II).

We used MICA and IMCCE data to calculate the visibility criteria ($C > C_{th}$) for the phase angles of 175, 174.5, 174, 173.5, 173, 172.5, 172, 171.5 and 171 degrees (Tables I and II). These angles correspond to the topocentric elongations of 4, 4.5, 5, 5.5, 6, 6.5, 7, 7.5, and 8 degrees respectively.

The calculated visibility criteria are shown in the last columns of both tables.

Table I **Table II** **Table III**

From Table I and II, we find that an elongation of 7.5 ° (±0.25°) is the lowest naked-eye-visibility limit. If the conjunction occurs when the Moon is near apogee, the Moon reaches to this elongation 15 hours and 11 minutes after conjunction. If the conjunction occurs when the Moon is near perigee, the Moon -due to its high velocity- attains this elongation 14 hours after conjunction. Moreover, when the Moon is near perigee, at an elongation of 7.5 °, the crescent has a width of $0.18'$ and its luminance is 9.5×10^6 nL. When the Moon is near apogee, it has a thinner width and more luminance, for the same elongation. For this reason, we get the same visibility limit in both cases.

After conjunction, the crescent width being thinner at the cusps, the Blackwell Contrast Threshold C_{th} is greater than C, so that cusps visibility becomes difficult, and the crescent seems shorter than 180 degrees[17].

Optically-aided visibility limit calculations

If stars were perfect point sources, the issue of background light pollution or moonlight could be resolved by simply increasing magnification. The background becomes darker while the star keeps the same brightness. We can even observe brighter stars in the daylight with suitably high magnification!

At night, as the skies get darker, we see more and more, fainter and fainter stars with the naked eye. With the telescope we see even more stars since the object is greatly magnified thus spreading out the background light but still gathering the star, which is a point source, into a relatively good point image. When looking at an extended object with a telescope, both the background and the object are spread out in a similar way so the actual contrast between them does not change but the eye can discern contrast better if the objects viewed are larger.

A faint extended object (e.g. a thin lunar crescent, a galaxy, or nebula) should be viewed with enough magnification so it appears several degrees across. A very thin crescent, it must be surrounded by a darker background if the eye is to detect contrast; keep in mind that when we raise the magnification 10 times means decreasing the surface brightness of both the crescent and background sky 100 times! Various magnifications should thus be tried to yield the best detection. At each magnification, considerable time should ideally be spent determining the quality of detection. Unfortunately, this is not the case for detecting the very young crescent moon as it appears only for few minutes before disappearing in the haze of twilight. With such situations even experienced observers would be in trouble (http://www.icoproject.org/icop/ram25.html).

Since magnifying the crescent enhances its visibility one must consider optically-aided visibility criteria. In Table III, we recalculate the first row of Table I by assuming the following: dark-adapted eyes, normal acuity, properly aimed and focused telescope, good optics, 200x, high elevation site, good transparency with minimum aerosols, and -most importantly- good experience!

In Table III, due to the 200x, we find that L drops from 2.4×10^7 to 600 nL, and L_B drops from 1.2×10^7 to 300 nL. So, C remains constant = 1 while C_{th} drops from 60 to 0.85 (Fig. 1) which leads to the interesting result: the lunar crescent may be seen even when the Moon is new!

Since we can spot the crescent at the moment of new Moon (*e.g.*, 2002 March 14 at 0205 UT), we can also spot it one minute earlier at 0204 UT when it is an 'old moon' and one minute latter at 0206 UT when it is a 'new moon', deducing that the Moon can pass from the 'old moon' phase to the 'new moon' phase without disappearing from sight. So, our calculations show that Danjon limit is not only less than 7˚, but it vanishes completely. Moreover, by assuming the 200x and by using Fig.1, we find that the dimensions[17] of the thin rectangle of light are about 233.2′ and 12′, concluding that the length of the arc of light of the lunar crescent at new moon, when the Moon is at its greatest inclination is about 7.1° instead of 180° *i.e.,* only 4% of the length of the normal few-days-old crescent moon.

Fig. 1

Now, if it is possible to spot the thin lunar crescent at new moon when the Moon is at its greatest inclination, why has nobody claimed this observation before? The answer is because the coincidence of excellent conditions of observations and ideal parameters - discussed above- rarely happens. However, our model predicts that such an observation will be made in the future, and Stamm's observation of 2004 Oct.13 was only the first step in spite of the conservative comment made by *Sky & Telescope* editors[21].

New Moon observations to be attempted in 2007

We urge the test of our model during the next moths of this year. The following are the dates of the new moon when the lunar elongation will be around 5° at the instant of new moon:

May 16 at 1929 UT, June 15 at 0314 UT, November 9 at 2304 UT and December 9 at 1741 UT.

Conclusions

Our lunar-first-visibility model, based on Blackwell's 1946 data, employs more extensive data than those used by all other visibility models. Our calculation shows that 7.5° (±0.25°) is the lowest naked eye visibility limit. It also shows that -with suitable magnification- the lunar crescent may be seen even when the lunar crescent is at new moon provided that the Moon is at its greatest inclination. The expected length of the lunar crescent is about 7.1° instead of 180°. Observing the lunar crescent at new moon is more probable before sunrise than after sunset due to dark adaptation.

Stamm spotted the thin crescent when the apparent Moon's elongation was only 6.4°. With better Equipment than those of Stamm and/or with better observation conditions than those of Mount Lemmon, Stamm's record would be broken again and again until arriving at our model prediction: the lunar crescent could be seen even when the lunar crescent is at new moon. In consequence, the Danjon limit is no longer valid.

We expect that our result will have considerable attraction for societies who are using Danjon limit as the basis for their religious calendars.

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Figure Captions

Figure1. This figure illustrates the possibility of spotting the thin lunar crescent at new moon when the Moon is at its greatest inclination *e.g.,* on 2002 March 14 at 0205 UT. From Table III, a telescopic magnification (200x) makes L_B dropp from 1.2×10^7 to 300 nL. On the 300 nL Blackwell curve we find that the value of C_{th} corresponding to the magnified crescent width W (12′) is equal to 0.85. However, C doesn't change due to magnification and keeps the same value which is equal to 1, that value fits our visibility criteria ($C > C_{th}$), leading to the interesting result: the lunar crescent can be seen at new Moon!

To calculate the visible crescent length, we use our relation deduced in a previous work[17]:

$$L = D - 2w\,(D + W)/2W,$$

where *L* represents the vertical component of the visible crescent length in arc minutes, *D* is the apparent diameter of the Moon in arc minutes, W is the width of the crescent in arc minutes and *w* is the largest invisible width in arc minutes *i.e.*, when $C_{th} = C$. From this figure we find that *w* is equal to 11.5′ (*w* corresponds to the point of intersection of the 300 nL curve and the line C_{th}=1. This figure also gives more accurate value for *w* than our previous approximate value[17]). The visible crescent length in degrees compared to the 180° normal crescent will be: L /D×180°.

Table I

Moon near apogee

(New Moon: Mach 14, 2002 at 02:05 UT)

(67°S, 106°W), elevation: 2000m

Obs. time UT Date	h	m	Ph.angle o	R ′	Mag	Elo n o	% ill	L_* (nl)	L (nl)	W ′	L_B (nl)	C_{th}	C	Vis $C > C_{th}$
14 3 2002	2	5	175	14.68	-4.43	4	0.19	4.3×10^8	2.4×10^7	0.06	1.2×10^7	60	1	no
14 3 2002	7	5	174.5	14.68	-4.48	4.5	0.23	3.7×10^8	2.1×10^7	0.07	7.5×10^6	43	2	no
14 3 2002	9	20	174	14.68	-4.54	5	0.27	3.4×10^8	1.9×10^7	0.08	4.5×10^6	41	3	no
14 3 2002	11	20	173.5	14.69	-4.6	5.5	0.32	3×10^8	1.7×10^7	0.1	2.6×10^6	38	6	no
14 3 2002	12	50	173	14.69	-4.65	6	0.37	2.7×10^8	1.5×10^7	0.11	1.5×10^6	33	9	no
14 3 2002	14	20	172.5	14.69	-4.7	6.5	0.42	2.5×10^8	1.4×10^7	0.12	8.5×10^5	31	15	no
14 3 2002	15	50	172	14.69	-4.75	7	0.48	2.3×10^8	1.3×10^7	0.14	4.8×10^5	32	26	no
14 3 2002	17	16	171.5	14.69	-4.81	7.5	0.55	2.1×10^8	1.2×10^7	0.16	2.7×10^5	37	43	yes
14 3 2002	18	40	171	14.69	-4.86	8	0.62	2×10^8	1.1×10^7	0.18	1.5×10^5	39	72	yes

Table II

Moon near perigee

(New moon: 23 11 2003, at 23:00 UT, Total Solar Eclipse)

Obs. time UT Date	h	m	Ph.angle o	R ′	Mag	Elo n o	Ill%	L_* (nl)	L (nl)	W ′	L_B (nl)	C_{th}	C	Vis $C > C_{th}$
24 11 2003	7	0	175	16.73	-4.42	4	0.19	3.3×10^8	1.8×10^7	0.06	1.2×10^7	60	0.5	no
24 11 2003	7	55	174.5	16.73	-4.48	4.5	0.23	2.7×10^8	1.5×10^7	0.08	7.5×10^6	37	1	no
24 11 2003	8	45	174	16.73	-4.54	5	0.27	2.5×10^8	1.4×10^7	0.09	4.5×10^6	33	2	no
24 11 2003	9	35	173.5	16.73	-4.59	5.5	0.32	2.3×10^8	1.3×10^7	0.11	2.6×10^6	35	4	no
24 11 2003	10	25	173	16.73	-4.65	6	0.37	2×10^8	1.1×10^7	0.13	1.5×10^6	27	6	no
24 11 2003	11	15	172.5	16.72	-4.7	6.5	0.43	1.9×10^8	1.1×10^7	0.14	8.5×10^5	26	12	no
24 11 2003	12	5	172	16.72	-4.75	7	0.48	1.8×10^8	1×10^7	0.16	4.8×10^5	25	20	no
24 11 2003	13	00	171.5	16.72	-4.81	7.5	0.55	1.7×10^8	9.5×10^6	0.18	2.7E5	28	34	yes
24 11 2003	13	48	171	16.72	-4.86	8	0.62	1.5×10^8	8.3×10^6	0.21	1.5×10^5	32	54	yes

Table III

Moon near apogee

(New moon: Mach 14, 2002 at 02:05 UT)

The lunar crescent could be seen when the Moon is at new moon

Obs. time UT Date	h	m	Ph.angle °	R ′	Mag	Elon °	% ill	L_* (nl)	L (nl)	W ′	L_B (nl)	C_{th}	C	Vis $C > C_{th}$
14 3 2002	2	5	175	14.68	-4.43	4	0.19	4.3×10^8	2.4×10^7	0.06	1.2×10^7	60	1	no
200×									0.6×10^3	12	0.3×10^3	0.85	1	yes

<u>Note (1): Abbreviations used in Table I, Table II and Table III</u>

Obs.time: Observation time in UT.

Ph.angle: phase angle in degrees.

R: Semi-diameter of the lunar disk.

Mag: lunar magnitude.

Elon: elongation in degrees (taking account of lunar parallax), DAZ=0°, and site elevation = 2000m.

%Ill: illuminated fraction of the lunar disk.

L_*: actual (extra-atmospheric) luminance of the moon.

L: The apparent (ground-observed) luminance of the moon (nL).

W: Topocentric width of the illuminated lunar disk in minutes of arc.

L_B: Twilight sky luminance (nL).

C_{th}: Blackwell Contrast Threshold C_{th}.

C: $(L - L_B)/ L_B$.

Vis: Visibility (yes or no).

<u>Note (2): Assumptions and applications adopted in table I, table II and Table III</u>

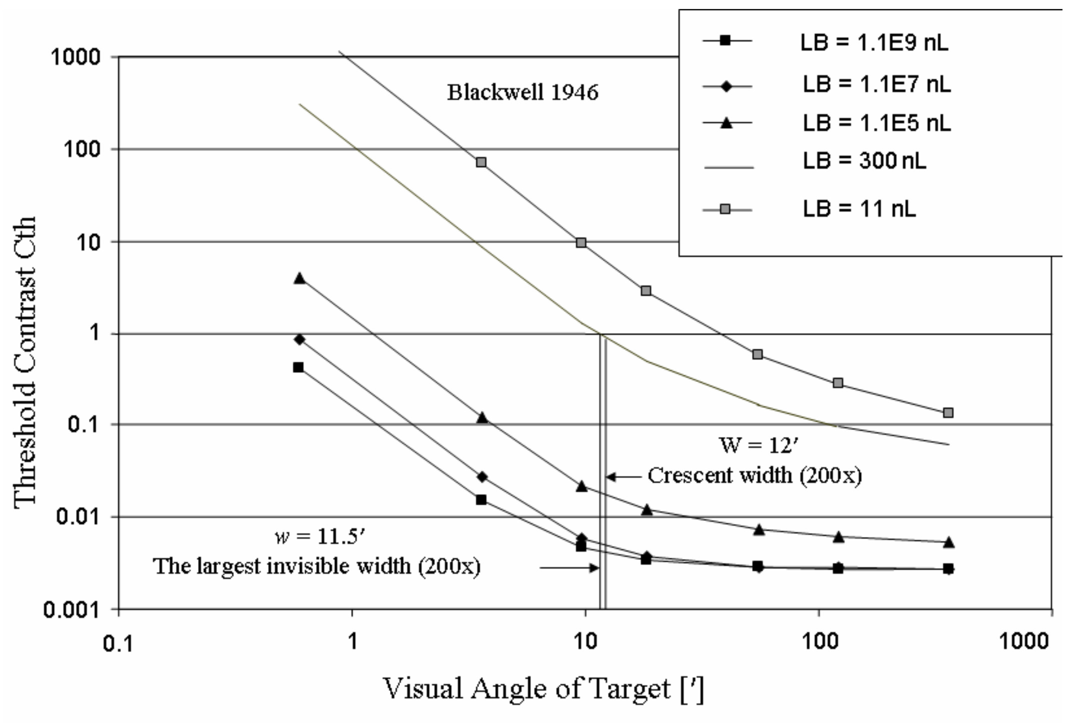
We consider DAZ = 0°; the perfect geometrical situation with the difference in azimuth between the Sun and the Moon at the moment of observation equals zero.

We consider the moon altitude = 2° at the moment of observation.

We apply our site elevation (2000m) to calculate L.

We use our measured L_B.

Figure1



LB = 1.1E9 nL
LB = 1.1E7 nL
LB = 1.1E5 nL
LB = 300 nL
LB = 11 nL
Blackwell 1946
1000
100
10
1
0.1
0.01
0.001
Threshold Contrast Cth
W = 12′
Crescent width (200x)
w = 11.5′
The largest invisible width (200x)
0.1
1
10
100
1000
Visual Angle of Target [′]