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Lunar Crescent Visibility

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SUMMARY

The visibility of the thin crescent Moon is an important problem for the calendars of many societies, both ancient and modern. With roughly 1×10^9 people of the Islamic faith following the Islamic calendar, this problem is likely to be the one (non-trivial) problem in astronomy that has the greatest impact on our modern world. In the past decade, great advances have been made in the observation and theory of crescent visibility. This paper reports recent observations and analyses. New records have been set for the youngest Moon, with confident sightings at 15·0 hr by John Pierce with unaided vision and at 12·1 hr by Jim Stamm with telescopic assistance. These records can be significantly broken under optimal conditions. Various prediction algorithms are tested with the 294 collected individual observations plus the 1490 observations from the five Moonwatches. The age and moonset-lag criteria are found to be poor, the altitude/azimuth criteria can make a confident prediction only one-quarter of the time, while the best predictor by far is the modern theoretical algorithm.

INTRODUCTION

Many ancient calendars were based on lunar months defined by the first (or last) sighting of the Moon near conjunction. As such, many problems in chronology require a detailed knowledge of the dates when the Moon can be seen. Historically, this has driven much astronomical research from Babylonian to medieval Arabic times. Several modern calendars of wide use are also based on lunar visibility, so the prediction and 'postdiction' of dates require a correct astronomical calculation for the calendar to have utility.

It turns out that crescent visibility is a tough problem, involving orbital calculations, lunar scattering, atmospheric scattering, and visual physiology. For a review of work up until the middle 1980s, see Ilyas (1994). In 1977, Bruin (1977) proposed that a modern theoretical approach be used to model the physics and physiology involved, and this has been followed up with a detailed working algorithm (Schaefer 1988). Starting in the late 1980s, five Moonwatches were organized to collect data systematically from across North America, with the result that detailed maps of visibility probabilities can be constructed from 1490 reports (Doggett & Schaefer 1994). Also collected were 252 individual observations recorded in the astronomical literature (Schaefer 1988; Doggett & Schaefer 1994). Thus the crescent problem is now on a sound theoretical and observational basis.

RECORD CRESCENTS

The sighting of young crescents has turned into a friendly competition among modern amateur astronomers. However, it is possible mistakenly to report a young Moon (Schaefer, Ahmad, & Doggett 1993), with an approximate error rate of 15 per cent (Doggett & Schaefer 1994). Thus claims of very young crescents must be closely scrutinized. Schaefer, Ahmad & Doggett (1993) find the reliable reports to be clearly distinguished by the observer's experience, the promptness of the report, and the observer's preparation, with consistency of the details with calculation an additional requirement.

In recent years, a number of record-breaking observations have been reported in detail to me (see Table I). The current record holders are John Pierce at 15·0 hr for unaided vision, and Jim Stamm at 12·1 hr for aided vision. (These ages are for the times of first sighting, while Table I gives the ages for the time of best visibility.) Let me now evaluate both sightings with the above criteria.

John Pierce observed from Collins Gap in eastern Tennessee, and has had previous experience at crescent watching. The first report that I have of this event is dated 1990 May 26, but a specific expedition of six observers is likely to get the date correct. The observers pre-calculated the position of the Moon (with respect to the sunset point) for a specific time so that they knew exactly where to look. Pierce spotted the Moon with the unaided eye, and the sighting was confirmed with a 12·5-inch telescope as seen by four people. Jan Kemp also saw the Moon with unaided vision, Clint Bach and Ed Byrd saw the crescent only through binoculars and the telescope, while Jim Golden and Travis Byrd could not see the Moon at all. The time of Pierce's first sighting is close to 1990 February 25 23:55 UT, for an age of 15·0 hr. The report contains no anomalies, so I accept it as correct.

Jim Stamm observed the crescent at 1996 January 21 00:57 UT (12·1 hr after conjunction) with an 8-inch telescope at 50×, from his home near Tucson, Arizona. Stamm has a long track-record of experience at searching for young crescent Moons (Doggett & Schaefer 1994). His observation was promptly reported to me and to the Internet news group SCI.ASTRO. Stamm's preparation was the most extensive of all attempts that I know of – with him pre-focusing the telescope the night before and pre-aiming the telescope with a timed observation of a star. I have checked in detail his report and can find no inconsistencies with calculation. For example, he correctly reported the arc-length of the crescent, and the progressive change of colours, and his times are as expected. In summary, Stamm's report well satisfies all my criteria for a reliable observation.

These records can be significantly broken for optimal conditions. For example, an observer at high altitude in northern California or Nevada should have been able to see Stamm's record-breaking Moon with the unaided eye. An optimal latitude (such that the Moon stands directly above the Sun) will improve visibility, as will a site with low humidity. For site selection, however, observers should strive for elevation to get above the low-lying atmospheric aerosols. Elevation can make or break a sighting attempt. For example, had Stamm been observing from an elevation of over 8000 feet above sea level, I calculate that the 12·1-hr-old Moon should have been detectable with the unaided eye. My other advice to observers is to set up early and to have some accurate means of knowing exactly where and when to look.

I have searched through the upcoming years for good opportunities to break the records substantially from the observatories on Cerro Tololo, Mauna Kea and La Palma. The two opportunities that I found were both for the top of Mauna Kea. On 1996 December 10, an 11·1-hr crescent is likely to be visible to the unaided eye. On 1997 February 7, a 13·3-hr crescent is likely to be seen with the unaided eye.

OBSERVATIONS

Earlier papers (Schaefer 1988; Doggett & Schaefer 1994) have collected 252 reports of crescent observations from astronomical sources. Since this time, I have collected 43 additional observations. McPartlan (1985) accounts for 25 reports, all made from the same site in the Sudan. The remainder have been collected by myself as private communications from the observer. The observations and associated parameters are tabulated in Table I. The columns are the same as in Doggett & Schaefer (1994).

The table also contains two corrections to the previous lists. Observation 117 had an incorrect time of conjunction and an incorrect age. Observation 44 should be deleted as unreliable, since Loewinger (1995) has found that the Schmidt observation was not made by Julius Schmidt (one of the greatest visual observers) but by his unskilled gardener Friedrich Schmidt in a casual observation. This reduces the total to 294 collected reports.

ALGORITHM TESTING

The many crescent visibility algorithms should be tested against all available data. Doggett & Schaefer (1994) used the extensive Moonwatch data involving 1490 reports from five nights to test 13 predictors. The conclusions were that (1) the ancient and medieval criteria were highly unreliable, (2) the altitude/azimuth relations have a reasonable accuracy, and (3) the modern theoretical algorithm has the lowest systematic error, the lowest average error, and the lowest maximum error by about a factor of 2 as compared with all other models. Their table IX quantifies these results.

The independent data set of 294 individual observations can be used for a further test of various algorithms. This investigation is presented below, with a summary in Table II.

Age

To a zeroth-order approximation, the crescent can only be sighted more than a day after conjunction. In ancient times, this was canonized as a rule that the Moon will be visible if its age is greater than 24 hr. Doggett & Schaefer (1994) showed that the average error for this criterion is 93° in longitude, while the maximum error is 206° of longitude, so that the whole world is in the 'zone of uncertainty'. This zone is the region over which an algorithm cannot produce a prediction with high confidence. The goal of crescent visibility research is to reduce this zone of uncertainty to a minimal size as measured in width of longitude.

With the 294 individual observations, a Moon as young as 15·0 hr has been sighted, while a Moon as old as 51·3 hr has been missed. A better test of the criterion is to create a histogram of the fraction of the time that the algorithm

TABLE I

Individual observations of crescent visibility

No.	Year	M	D	E/M	JD (conj.)	Observer	Vis.	Lat.	Long.	Alt.	RH	k_a	ARCL	ARCV	DAZ	Age	Lag	R±DR	Y/N	(Sig)
44	1871	9	14	M	2404685.300	F.Schmidt	Unreliable observation…													
117	1971	3	27	E	2441037.308		V	51.0	0.0	100	70	18	14.7	14.4	3.0	23.8	93	1.0±0.3	Y	(3)
253	1983	11	5	E	2445643.432	M.McPartlan	I	15.6	35.6	1100	30	8	9.3	8.8	3.1	17.0	37	−0.7±0.3	Y	(2)
254	1983	11	6	E	2445643.432	M.McPartlan	V	15.6	35.6	1100	30	8	21.5	19.0	10.3	41.3	83	2.7±0.1	Y	( )
255	1983	12	5	E	2445673.019	M.McPartlan	I	15.6	35.6	1100	30	7	13.4	12.1	5.8	27.1	54	1.0±0.3	N	(−4·)
256	1984	1	3	E	2445702.709	M.McPartlan	I	15.6	35.6	1100	30	7	5.6	4.2	3.7	10.3	19	−2.8±0.3	Y	(9)
257	1984	1	4	E	2445702.709	M.McPartlan	V	15.6	35.6	1100	30	7	16.5	15.5	5.8	34.9	70	2.0±0.2	Y	( )
258	1984	2	2	E	2445732.491	M.McPartlan	I	15.6	35.6	1100	30	8	8.7	7.4	4.5	16.0	32	−1.2±0.4	Y	(3)
259	1984	2	3	E	2445732.491	M.McPartlan	V	15.6	35.6	1100	30	8	18.9	18.5	4.0	40.4	80	2.5±0.1	Y	( )
260	1984	3	3	E	2445762.272	M.McPartlan	I	15.6	35.6	1100	30	10	10.9	10.3	3.6	21.6	43	0.1±0.3	N	(−0)
261	1984	3	4	E	2445762.272	M.McPartlan	V	15.6	35.6	1100	30	10	21.2	21.1	1.8	45.9	88	2.8±0.1	Y	( )
262	1984	4	2	E	2445792.008	M.McPartlan	I	15.6	35.6	1100	30	11	13.5	13.3	2.0	28.1	55	1.1±0.3	N	(−4·)
263	1984	4	3	E	2445792.008	M.McPartlan	V	15.6	35.6	1100	30	11	24.4	24.4	0.2	52.3	102	3.1±0.1	Y	( )
264	1984	5	2	E	2445821.657	M.McPartlan	V	15.6	35.6	1100	30	18	17.8	17.8	1.1	36.8	77	1.9±0.2	Y	(8)
265	1984	5	31	E	2445851.201	M.McPartlan	I	15.6	35.6	1100	30	23	12.3	12.3	1.1	23.7	55	−0.1±0.5	Y	(0)
266	1984	6	1	E	2445851.201	M.McPartlan	V	15.6	35.6	1100	30	23	24.9	24.7	3.3	48.0	112	2.9±0.2	Y	( )
267	1984	6	30	E	2445880.639	M.McPartlan	V	15.6	35.6	1100	30	23	20.9	20.3	5.0	37.5	92	2.3±0.2	Y	( )
268	1984	7	29	E	2445909.994	M.McPartlan	V	15.6	35.6	1100	30	22	17.2	16.4	5.1	28.9	72	1.5±0.3	Y	(5)
269	1984	8	27	E	2445939.310	M.McPartlan	I	15.6	35.6	1100	30	24	13.1	12.5	3.8	20.9	52	0.0±0.5	N	(−0)
270	1984	8	28	E	2445939.310	M.McPartlan	V	15.6	35.6	1100	30	24	26.7	23.3	13.2	45.1	98	2.9±0.2	Y	( )
271	1984	9	25	E	2445968.633	M.McPartlan	I	15.6	35.6	1100	30	18	8.3	8.1	1.7	12.5	33	−1.8±0.5	Y	(4)
272	1984	9	26	E	2445968.633	M.McPartlan	V	15.6	35.6	1100	30	18	21.7	18.6	11.2	36.9	77	2.4±0.2	Y	( )
273	1984	10	25	E	2445998.006	M.McPartlan	I	15.6	35.6	1100	30	46	15.8	13.5	8.3	27.4	58	−0.3±0.7	Y	(0)
274	1984	10	26	E	2445998.006	M.McPartlan	V	15.6	35.6	1100	30	9	29.4	24.2	16.8	51.7	107	3.4±0.1	Y	( )
275	1984	11	23	E	2446027.457	M.McPartlan	I	15.6	35.6	1100	30	8	9.3	7.6	5.3	16.3	34	−1.0±0.3	Y	(3)
276	1984	11	24	E	2446027.457	M.McPartlan	V	15.6	35.6	1100	30	8	22.5	19.3	11.5	40.7	88	2.8±0.1	Y	( )
277	1984	12	23	E	2446056.992	M.McPartlan	V	15.6	35.6	1100	30	7	15.2	13.4	7.1	28.0	61	1.5±0.2	Y	(7)
278	1990	2	25	E	2447947.872	J.Pierce+	V(V)	35.6	−83.5	5000	70	8	8.6	8.6	−0.5	14.8	42	−0.7±0.3	N	(−2·)
279	1990	2	25	E	2447947.872	C.Bach+	I(V)	35.6	−83.5	5000	75	9	8.6	8.6	−0.5	14.8	42	−0.8±0.4	Y	(2)
280	1990	2	25	E	2447947.872	J.Golden+	I(I)	35.6	−83.5	5000	75	9	8.6	8.6	−0.5	14.8	42	−0.8±0.4	Y	(2)
281	1990	5	24	E	2448035.992	C.Bach+	I(V)	35.6	−83.5	5000	70	15	9.0	9.0	0.3	13.2	51	−1.1±0.4	Y	(3)
282	1990	11	19	E	2448212.878	B.Schaefer	V	39.0	−76.8	260	60	13	27.9	15.2	23.5	61.5	90	2.6±0.3	Y	( )
283	1990	12	18	E	2448242.683	B.Schaefer	V	39.0	−76.8	260	60	12	19.0	13.8	13.2	42.1	82	1.7±0.3	Y	(6)
284	1991	3	17	E	2448331.841	B.Schaefer	V	39.0	−76.8	260	60	19	21.5	21.5	0.8	39.8	111	2.5±0.2	Y	( )
285	1991	5	15	E	2448390.692	B.Schaefer	V	39.0	−76.8	260	60	25	25.8	23.0	11.7	44.5	135	2.8±0.2	Y	( )
286	1991	9	7	M	2448507.960	M.Hedges	V(V)	50.4	−4.4	400	85	33	17.1	12.9	11.2	−29.9	83	0.2±0.6	Y	(0)
287	1992	3	5	E	2448686.058	B.Schaefer	V	39.0	−76.8	260	60	17	16.9	16.9	−0.5	34.4	87	1.7±0.3	Y	(7)
288	1992	4	2	M	2448715.710	K.Krisciunas	I(I)	19.8	−155.5	13700	65	1	8.1	8.1	0.8	−13.2	34	0.2±0.2	N	(−0.5)
289	1995	1	1	E	2449718.956	P.Schwaar	I(V)	33.0	−106.0	4000	50	5	9.1	9.1	−0.3	13.6	47	−0.2±0.3	Y	(1)
290	1995	6	28	E	2449896.535	B.Schaefer	V	−30.1	−71.0	9100	50	1	11.0	11.0	−0.1	21.7	55	1.2±0.1	Y	(9)
291	1996	1	19	M	2450103.035	J.Stamm	V(V)	32.4	−111.0	2800	45	6	14.3	13.2	5.6	−23.0	67	1.4±0.2	Y	(7)
292	1996	1	20	E	2450103.035	J.Stamm+	I(V)	32.4	−111.0	2800	50	6	8.8	8.5	−1.9	12.3	43	−0.6±0.3	Y	(2)
293	1996	1	20	E	2450103.035	P.Schwaar+	I(V)	32.8	−113.2	850	50	7	8.8	8.6	−1.8	12.4	43	−0.8±0.3	Y	(2)
294	1996	1	20	E	2450103.035	D.Patchick+2	I(V)	34.1	−118.3	1740	50	11	9.0	8.8	−1.5	12.7	45	−1.0±0.4	Y	(3)
295	1996	1	20	E	2450103.035	D.Patchick+7	I(I)	34.1	−118.3	1740	50	11	9.0	8.8	−1.5	12.7	45	−1.0±0.4	Y	(3)

TABLE II

Summary of algorithm tests

Algorithm	Moonwatch data		Individual data	
	Average error	Maximum error	Biased?	Total width
Age	93°	206°	Yes	238°
Moonset lag	66°	144°	Yes	150°
Altitude/azimuth	40°	70°	Yes	94°
Modern theoretical	11°	23°	No	59°

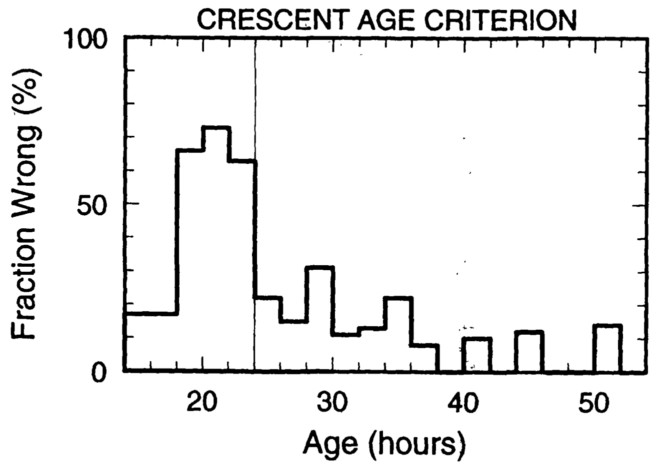

FIG. 1. A test of the age criterion. This histogram shows the fraction of the 294 collected individual observations for which the age criterion (i.e. the Moon should be visible if its age is > 24 hr) predicts the wrong answer. An unbiased model should never show bins with > 50 per cent error fractions. The horizontal range of the plot covers the middle 90 per cent of all 294 data points, so that the width of the histogram is an indication of the accuracy of the model. In the case of the 24-hr threshold, three bins are > 50 per cent wrong and the predictions are frequently incorrect over all considered ages. In other words, age is a bad predictor of lunar visibility.

produces the wrong result as a function of age (see Fig. 1). If the algorithm were perfect (i.e. the Moon suddenly turned on at an age of 24 hr), then the histogram should be close to zero per cent wrong for all ages. If the algorithm were good, then the fraction wrong would be significant only near the threshold (in this case, an age of 24 hr). An unbiased criterion would have a histogram that always has less than 50 per cent incorrect predictions, as otherwise a slightly shifted criterion can be constructed with improved reliability. The histograms are plotted with the complete range along the horizontal axis covering the middle 90 per cent of the range from all 294 observations, so that the compactness of the histogram is a measure of the accuracy of the criterion. With this, we can now interpret Fig. 1. The histogram is > 50 per cent for ages from 18 to 24 hr, indicating that the age criterion is sharply biased. For crescent ages from 18 to 30 hr, a coin flip is a reliable as the age threshold. The age criterion is violated significantly over the entire range in question, so that I conclude that it can never give a reliable answer.

Let us now estimate the width of the zone of uncertainty as measured in degrees of longitude. The root-mean-square deviation for the histogram is

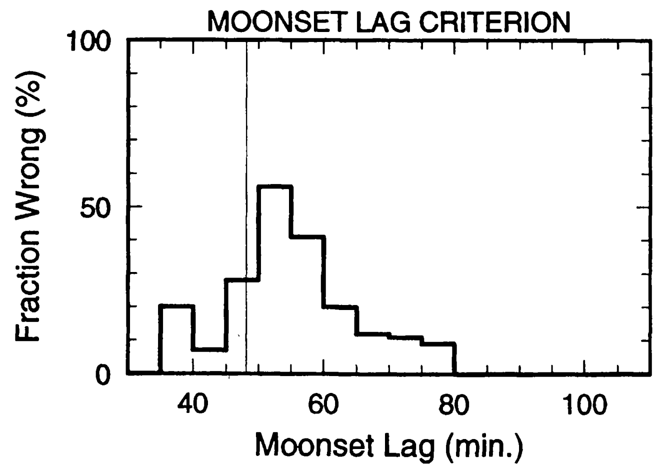

FIG. 2. A test of the moonset-lag criterion. The Babylonians advanced a claim that the Moon would be visible if it set more than 48 min after the Sun. This histogram shows the fraction of wrong answers yielded by this method as a function of the moonset lag. One of the bins is > 50 per cent wrong, indicating that the Babylonian criterion is biased. The width of the histogram translates into a zone of uncertainty that covers the entire world, which is to say that this method can never yield a confident prediction.

7·9 hr, which corresponds to 119° of longitude. So even just the 1σ total width of the zone of uncertainty for the age criterion is 238°. The entire world is within the approximately 1·5σ zone of uncertainty, which is to say that this criterion is very poor.

Moonset Lag

The ancient Babylonians developed a rule that the Moon should be visible if the time that it sets lags by more than 48 min after sunset. For the Moonwatches, this predictor yields an average error of 66° in longitude and a maximum error of 144° in longitude (Doggett & Schaefer 1994). As such, any prediction from this Babylonian criterion must be unreliable.

With the 294 individual observations, we see the same pattern of unreliability. Crescents with moonset lags as short as 35 min have been seen, while crescents with lags as large as 75 min have been invisible. Again, a better test of the criterion is to create a histogram displaying the fraction of times that the prediction is wrong as a function of moonset lag (see Fig. 2). The bin for 50–55 min lag has greater than 50 per cent error, indicating that the moonset-lag criterion is biased. The broad distribution (compared with the middle 90 per cent range plotted) shows this model to be poor. A quantitative measure is provided by the root-mean-square deviation of the histogram, which equals 10·0 min. As the sky turns at a rate of 1° every 4 min, the 10·0-min uncertainty translates into a 2°·5 shift in lunar position. With an average lunar motion of 0°·5 per hour with respect to the Sun, it will take the Moon 5·0 hr to traverse the sky from the optimal position to the 1σ position. While this happens, the Earth has turned 75° of longitude. So the total 1σ width of the zone of uncertainty is 150° of longitude. The entire world is always within the 2·4σ zone of uncertainty, which is to say that the criterion is poor.

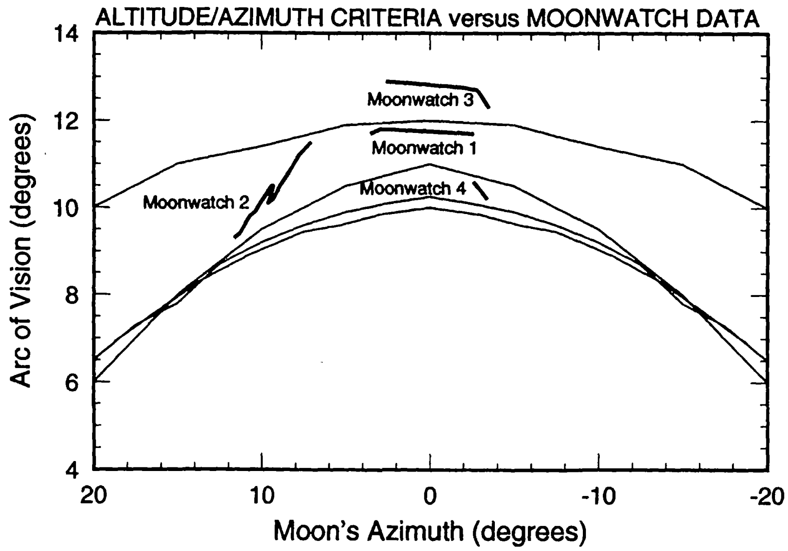

FIG. 3. Altitude/azimuth criteria versus the Moonwatch data. The altitude of the Moon above the Sun (the arc of vision) must exceed some threshold value (as a function of the relative azimuth of the Sun and Moon) for the crescent to be sighted. Historically, the thresholds have all been derived from one set of data, yet, even so, various investigators derive widely disparate relations. In any case, the well-observed Moonwatches show that the actual thresholds are widely variable from site to site and month to month. The reason for the failing of all altitude/azimuth criteria is that they do not account for the large variations in the haziness of the air as a function of the seasons, latitude, elevation and relative humidity.

Altitude/azimuth

The relative altitudes and azimuths of the Sun and Moon are some of the more important parameters for determining the brightness of the crescent and the brightness of the sky. As such, a plot of the relative altitudes of the Sun and Moon (the arc of vision) as a function of the relative azimuth can be divided into regions of visibility and invisibility. This connection with the physics of the problem guarantees that an altitude/azimuth formulation will be superior to the age or moonset lag formulation. The problem then comes in calibrating the dividing line. Historically, this has always been done with empirical data from one particular set of observations, primarily taken by Julius Schmidt at the Athens Observatory in the 1800s. Unfortunately, many researchers have analysed these same data with widely varying conclusions. Figure 3 shows four published criteria from Fotheringham (1910), Maunder (1911), Ilyas (1984), and Ilyas (1988). For the Moon exactly over the Sun, the threshold arc of vision is $12^\circ\!\cdot\!0$ for Fotheringham, $11^\circ\!\cdot\!0$ for Maunder, $10^\circ\!\cdot\!0$ for the first Ilyas claim, and $10^\circ\!\cdot\!3$ for the second Ilyas claim. This wide dispersion of researchers analysing exactly the same data is a strong indication that the altitude/azimuth criteria have significant problems.

The threshold altitudes (as a function of azimuth) for four Moonwatches are also drawn in Fig. 3. As the measurement uncertainty is much smaller than the separation, we immediately see that the threshold varies widely from lunation to lunation and even from state to state. This is not surprising, since the haziness of the air changes drastically with time of year and with location. Nevertheless, the altitude/azimuth relations are applied blindly, regardless of

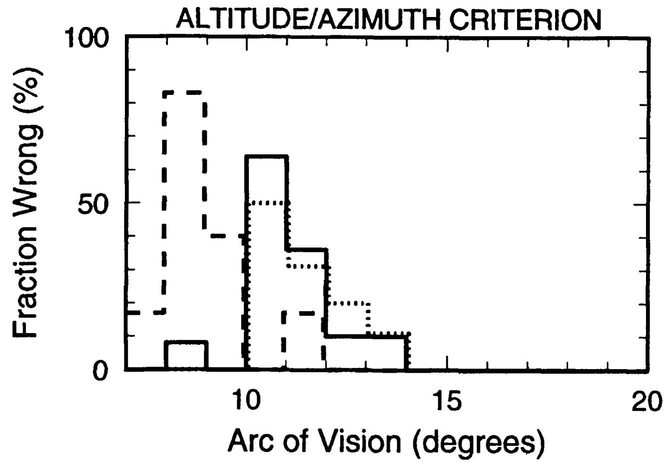

FIG. 4. Tests of the altitude/azimuth criteria. The histogram presents the fraction of wrong answers as a function of the arc of vision for three azimuth ranges: $< 5°$ (solid line), 5°–10° (dotted line) and $> 10°$ (dashed line). the presence of bins > 50 per cent wrong indicates a biased criterion. The altitude/azimuth formulation is a significant improvement over the two ancient criteria (see Figs 1 and 2), yet nevertheless the width of the histogram implies that the zone of uncertainty covers three-quarters of the world.

whether a clear winter sky in the Arizona desert or turbid summer air hanging over a Louisiana swamp is being considered. A quantitative analysis of the Moonwatch data shows typical average errors of 40° in longitude and maximum errors of 70° in longitude (Doggett & Schaefer 1994).

For the 294 individual observations, for the azimuth of the Moon within 5° of the azimuth of the Sun, the crescent has been sighted with the unaided eye with an arc of vision as low as 8°·6, while it has been missed for values as high as 13°·3. Figure 4 shows a histogram of the fraction of wrong predictions for the Maunder criterion for three separate ranges of azimuth separation ($< 5°$, 5°–10°, and $> 10°$). Again, the presence of > 50 per cent wrong bins indicates significant biases. The spread of wrong answers covers a substantial portion of the whole range. To be more quantitative, the root-mean-square deviations of the histograms are 1°·1, 1°·0 and 1°·1 respectively. To translate this into a longitudinal uncertainty, we can follow the same path as used in the previous section, but must divide by the cosine of the observer's latitude to account for the motion of the Moon not being perpendicular to the horizon. For temperate latitudes, a 1°·1 uncertainty in the threshold arc of vision translates into a 47° 1σ uncertainty in longitude. The total width of the zone of uncertainty will then be 94°, a poor value that is none the less substantially better than provided by the ancient criteria. The whole world fits into the $3{\cdot}8\sigma$ zone of uncertainty. For any lunation, the altitude/azimuth criteria cannot make a confident prediction for roughly three-quarters of the world.

Modern theoretical algorithm

The modern theoretical algorithm is based on the idea of Bruin (1977) that the physical and physiological processes involved can be closely modelled. This algorithm is extensively described by Schaefer (1988, 1990, 1993) and

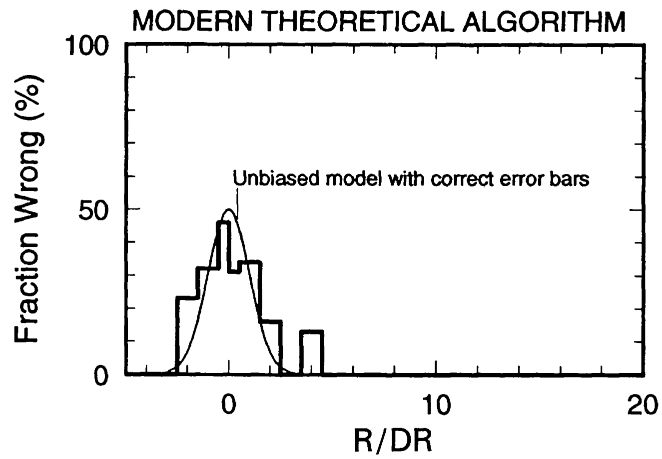

FIG. 5. A test of the modern theoretical algorithm. The plot is a histogram of the fraction of wrong answers as a function of R/DR. The lack of bins with > 50 per cent wrong answers and the centring near zero show the model to be without bias. The small width compared with the other histograms emphasizes that this model is the most accurate. The 1σ error corresponds to 29° of longitude.

Doggett & Schaefer (1994). A significant advantage of this model over all other predictors is that the atmospheric haziness is directly calculated and applied, so that the greatly different visibilities in Arizona and Louisiana will be correctly accounted for. The extinction coefficients are calculated from seasonal, latitudinal and elevation correlations corrected for the seasonal average evening relative humidity. The aerosol scattering, Rayleigh scattering and ozone absorption components are handled separately owing to their distinct vertical structures. The handling of the atmosphere on a site-by-site and month-by-month basis suggests that the modern theoretical algorithm will be a substantial improvement over the old altitude/azimuth relations.

The modern theoretical algorithm calculates the R-parameter, which must be positive for visibility to be possible. R is the logarithm of the actual brightness of the crescent divided by the detection threshold brightness at the optimal time during the twilight. The algorithm also calculates DR, which is the estimated 1σ uncertainty. The quantity R/DR is a measure of the confidence in the visibility, such that a large positive value implies easy visibility while a value near zero implies a location near the middle of the zone of uncertainty. Doggett & Schaefer (1994) test the algorithm with the Moonwatch data and find a mean error of 11° of longitude and a maximum error of 23° of longitude.

The 294 individual observations can also be used to test the modern theoretical algorithm. The crescent has been sighted for a value of R/DR as negative as $-2{\cdot}5$, while it was missed for a value as positive as $+4{\cdot}0$. Figure 5 shows a histogram of the fraction of wrong predictions as a function of R/DR. Since this quantity is the value of R divided by its calculated 1σ error, an unbiased model with correctly calculated uncertainties should produce a histogram consistent with a Gaussian that peaks at $R/DR = 0$ with a maximum of 50 per cent and a sigma of unity (see Fig. 5). The root-mean-square deviation of the histogram is 1·4, somewhat larger than the desired 1·0, which shows that the model calculations are somewhat optimistic in the accuracy. [This difference is not worrisome, as it arises entirely from the two missed crescents with R/DR equalling 3·8 and 4·0. The rate of positive errors for experienced observers (see Doggett & Schaefer 1994) has been measured to be > 1 per cent, so that some broadening of the distribution owing to observer errors is expected. Since the calculated DR does not include observer error, the results are still consistent.] The centroid of the histogram is at $0{\cdot}2 \pm 0{\cdot}2$ and all bins are less than 50 per cent, which indicates that the model has no substantial biases. To evaluate the average size of the zone of uncertainty, we need to know the rate of change of R and the mean value of the 1σ error for situations near the threshold. For a large number of near-threshold conditions, I find that a change of 60° in longitude (with all else held constant) produces an average change of $0{\cdot}88 \pm 0{\cdot}06$ in R. For the observations with $-1 < R < 1$, the average DR value is $0{\cdot}31 \pm 0{\cdot}01$, which for consistency should be multiplied by the empirical factor of 1·4 from the width of the histogram in Fig. 5. Thus the 1σ uncertainty in R is 0·43, which is 49 per cent of the change in R made by 60° of longitude near threshold. The 1σ value then corresponds to 29° in longitude, and a total width of the zone of uncertainty of 59°.

CONCLUSIONS

I have collected a total of 294 individual observations of crescent visibility from the astronomical literature. My conclusions are as follows. (1) The current world record for a young Moon with no telescopic assistance is 15·0 hr for John Pierce. (2) The current world record for a young Moon with telescopic assistance is 12·1 hr for Jim Stamm. (3) These records can be substantially broken under optimal conditions. (4) Good opportunities to break the record are on 1996 December 10 and 1997 February 7 from the top of Mauna Kea. (5) Observers seeking to break the record are strongly advised to get to the highest possible elevation and to devise some means to know exactly where and when to look. (6) The lunar age criterion is virtually useless. (7) The moonset-lag criterion can never produce a confident prediction. (8) The various altitude/azimuth relations can only provide a confident prediction one out of four times. (9) The modern theoretical algorithm is by far the best model by all measures.

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